Boundary effects in extended dynamical systems

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Abstract

In the framework of spatially extended dynamical systems, we present three examples in which the presence of walls lead to dynamic behavior qualitatively different from the one obtained in an infinite domain or under periodic boundary conditions. For a nonlinear reaction-diffusion model we obtain boundary-induced spatially chaotic configurations. Nontrivial average patterns arising from boundaries are shown to appear in spatiotemporally chaotic states of the Kuramoto-Sivashinsky model. Finally, walls organize novel states in simulations of the complex Ginzburg-Landau equation.

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1 Introduction.

Most of our current knowledge of spatiotemporal chaos comes from its analysis in the infinite size limit or from simulations in finite domains with periodic boundary conditions[1]. However, different geometries or boundary conditions may lead to substantially different dynamical behavior. We will exemplify this assertion by showing results from three different extended dynamical systems in which the dynamics is strongly influenced by the boundaries.

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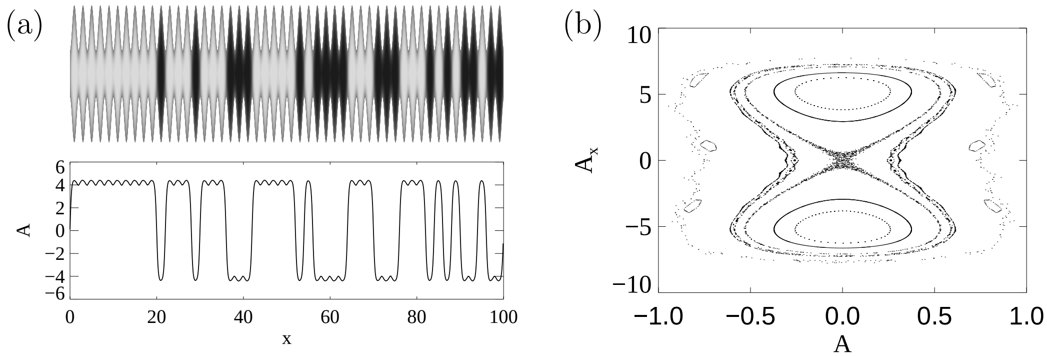

Fig. 1. (a) Top: two-dimensional configuration of our reaction-diffusion equation starting from random initial conditions. Bottom: Amplitude at the center of the domain. (b) Stroboscopic Poincaré map of the phase space of an approximation to our model equation.

2 Frozen spatial chaos induced by boundaries

We first show how the presence of nontrivial boundaries can induce the appearance of *spatial chaos* in a system for which no chaotic behavior is found neither in the infinite size limit nor with purely periodic boundary conditions[2]. The model we consider is a nonlinear diffusion equation of the Fisher-Kolmogorov type: $\partial_t A = \nabla^2 A + A - A^3$. The real quantity $A = A(x, y, t)$ is a twodimensionally extended field. When solved in doubly periodic integration domains, regions in which $A \approx \pm 1$ form, grow, and compete until one of the two phases takes over the whole system. When solved in regions such that Dirichlet (that is $A = 0$) conditions are applied in lateral boundaries which are not straight but undulating (see Fig. 1) the result is different: Frozen states in which the $A = +1$ and $A = -1$ phases alternate in space become stable and attract most of the initial conditions. The alternation of the two phases is random and produces static but spatially chaotic configurations. The justification of the 'chaotic' adjective can be done with different dynamical systems tools. For example Fig. 1b shows a Poincaré map of some of the spatial configurations obtained from an approximation to our model equation. KAM tori and other fractal structures are evident, in direct analogy with the classical picture of Hamiltonian systems with chaotic time trajectories. Theoretical arguments can be developed to show that the effect of the spatially undulated boundaries on the spatial pattern is similar to time-periodic parametric forcing in common temporal dynamical systems, from which the above chaotic phenomenology can be understood. Further details are given in [2]

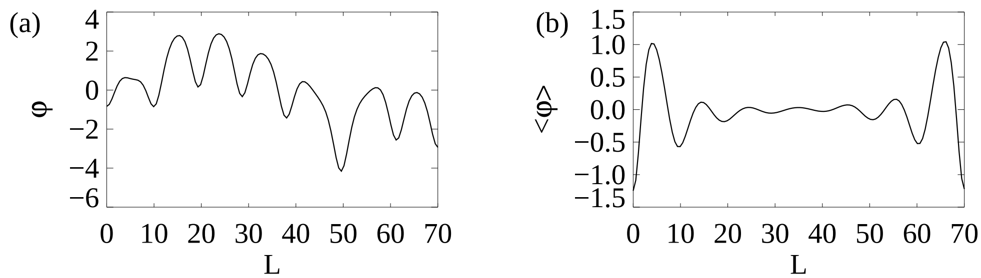

Fig. 2. (a) A characteristic configuration of the one-dimensional Kuramoto-Sivashinsky equation with stress-free boundary conditions. (b) The time averaged field.

3 Average patterns of spatiotemporal chaos.

Chaotic pattern dynamics in many experimental systems [3,4] show structured time averages. In this second Section, we suggest that simple universal boundary effects underlie this phenomenon and exemplify them with the Kuramoto-Sivashinsky equation in a finite domain. Figure 2b shows a structured average pattern for the Kuramoto-Sivashinsky equation in one dimension ($\dot{\varphi} = -\partial_x^2\varphi - \partial_x^4\varphi + (\partial_x\varphi)^2$) with stress-free boundary conditions (null first and third spatial derivatives at the boundaries). In contrast, the strong fluctuations of the instantaneous field are also shown in Fig. 2a for comparison. As in the experiments, the average pattern recovers the symmetries which are respected by both the equation and the boundary conditions (in this case left-right symmetry) locally broken in the chaotic field. The amplitude is strongest at the boundaries and decays through the center of the average pattern. The strength of the oscillations in the average pattern follows a $L^{-1/2}$ dependence on system size. Plateaus in the average-pattern wavenumber as a function of the system size are observed[5]. Most of these observations are also found in experimental systems[3,4] for which the Kuramoto-Sivashinsky equation does not apply, thus indicating its generic, mainly geometrical, origin: what is relevant for these phenomena to appear is the occurrence of strong enough chaotic fluctuations in the presence of non-trivial boundaries.

4 The complex Ginzburg-Landau equation in bounded domains.

In our third example, the effect of a finite geometry on the two-dimensional complex Ginzburg-Landau equation (in the Benjamin-Feir stable regime[1]) is addressed. Boundary conditions induce the formation of novel states. For

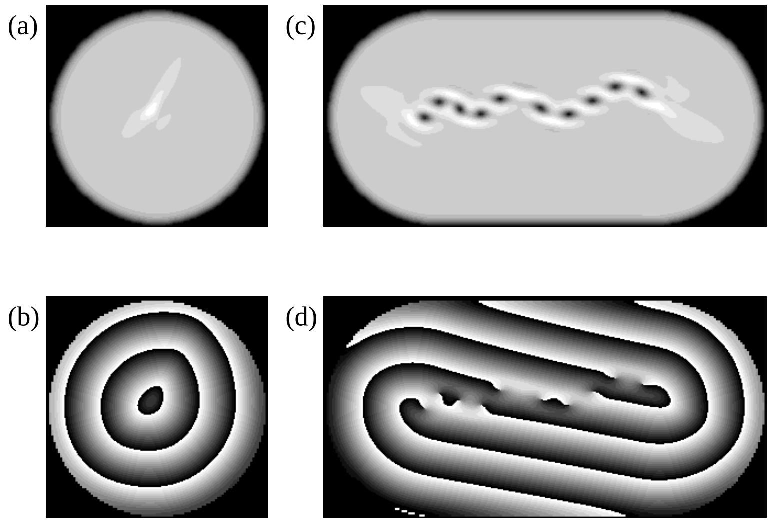

Fig. 3. Frozen structures for the complex Ginzburg-Landau equation in the Benjamin-Feir stable regime under null Dirichlet boundary conditions in different domains. Top raw: modulus; bottom raw: phase

example target like-solutions (Fig. 3a-b) appear as robust solutions under Dirichlet boundary conditions, whereas they are not observed under periodic boudary conditions. Dirichlet boundary conditions play a double rôle as sources (or sinks) of defects and as emitters of plane waves. The interplay between these two properties of the boundaries gives rise to interesting behavior[6,7]. In a square, walls emit waves that develop shock lines when they cross. Spiral defects form chains anchored by these shock lines. In circular domains, however, the emission is definitively dominated by the internal spiral defects. In a stadium geometry (Figs. 3c-d), typically a chain of defects links the centers of the circular regions. Synchronization of boundary wave emission is also found[6,7]. Most of these phenomena can be understood from the emission properties of Dirichlet walls[7].

5 Conclusions.

We have shown three examples where the boundary conditions play a key rôle in extended dynamical systems. Further work is needed to clarify the degree of universality of these results and to find general properties of different kinds of boundaries.

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